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The Crank-Nicolson temporal discretization scheme can be viewed as summation of implicit and explicit Euler discretization schemes.

For any variable ϕ, let assume the linear system for the partial differential equation is written as:

$$\frac{\partial \phi}{\partial t} = \mathbf{S}(\phi), \tag{1}$$

where $\mathbf{S}$ is the spatial discretization operator.

The temporal derivatives can discretized as :

Implicit Euler

$$\frac{\partial \phi^{t+\frac{1}{2}}}{\partial t} = \mathbf{S}(\phi^{t+\frac{1}{2}})$$

$$\frac{\phi^{t+\frac{1}{2}} - \phi^{t}}{(\Delta t/2)} = \mathbf{S}(\phi^{t+\frac{1}{2}}) \tag{2}$$

Explicit Euler

$$\frac{\partial \phi^{t+1}}{\partial t} = \mathbf{S}(\phi^{t+\frac{1}{2}})$$

$$\frac{\phi^{t+1} - \phi^{t+\frac{1}{2}}}{(\Delta t/2)} = \mathbf{S}(\phi^{t+\frac{1}{2}}) \tag{3}$$

Adding equations 2 and 4, we get

$$\frac{\phi^{t+\frac{1}{2}} - \phi^{t}}{(\Delta t/2)} + \frac{\phi^{t+1} - \phi^{t+\frac{1}{2}}}{(\Delta t/2)} = 2 * \mathbf{S}(\phi^{t+\frac{1}{2}})$$

$$\frac{\phi^{t+1} - \phi^{t}}{(\Delta t)} = \mathbf{S}(\phi^{t+\frac{1}{2}})$$

Final form of the discretization becomes:

$$\frac{\phi^{t+1} - \phi^{t}}{\Delta t} = \frac{1}{2}\left[\mathbf{S}(\phi^{t+1}) + \mathbf{S}(\phi^{t})\right] \tag{4}$$

The temporal term is similar to the implicit Euler scheme, and key difference is in semi implicit treatment of the spatial operators.