u_g gas/carrier velocity

u_p average particle velocity during the time step

p momentum

Momentum change of a particle (assuming true, average particle velocity u_p is known)

$$\Delta p_p = Sp_c(u_g - u_p)\Delta t + Sp_{nc}(u_g - u_p)\Delta t + Su_c\Delta t + Su_{nc}\Delta t$$

Momentum change of gas/carrier fluid

$$\Delta p_g = -Sp_c(u_g - u_p)\Delta t - Su_c\Delta t$$

Case 1: ($Sp_{nc} = 0$)

$$\Delta p_p = Sp_c(u_g - u_p)\Delta t + Su_c\Delta t + Su_{nc}\Delta t$$
$$-Sp_c(u_g - u_p)\Delta t - Su_c\Delta t = -\Delta p_p + Su_{nc}\Delta t$$

From this we get

$$\Delta p_g = -Sp_c(u_g - u_p)\Delta t - Su_c\Delta t = -\Delta p_p + Su_{nc}\Delta t = -(\Delta p_p - Su_{nc}\Delta t)$$

Case 2: ($Sp_{nc} \neq 0$)

$$\Delta p_p = Sp_c(u_g - u_p)\Delta t + Sp_{nc}(u_g - u_p) + Su_c\Delta t + Su_{nc}\Delta t$$
$$\Delta p_p - Su_{nc}\Delta t - Su_c\Delta t = (Sp_c + Sp_{nc})(u_g - u_p)\Delta t$$
$$\frac{Sp_c}{Sp_c + Sp_{nc}}(\Delta p_p - Su_c\Delta t - Su_{nc}\Delta t) = Sp_c(u_g - u_p)\Delta t$$

From this we get

$$\Delta p_g = -Sp_c(u_g - u_p)\Delta t - Su_c\Delta t = -\frac{Sp_c}{Sp_c + Sp_{nc}}(\Delta p_p - Su_c\Delta t - Su_{nc}\Delta t) - Su_c\Delta t$$