In Eulerian-Eulerian approach the momentum equations for gas and solid phases are usually something like:

$$\frac{\partial}{\partial t}(\alpha_g \rho_g \boldsymbol{u}_g) + \nabla \cdot (\alpha_g \rho_g \boldsymbol{u}_g \boldsymbol{u}_g) = -\alpha_g \nabla p + \nabla \cdot \boldsymbol{\tau}_g + \alpha_g \rho_g \boldsymbol{g} + K_{gs}(\boldsymbol{u}_s - \boldsymbol{u}_g) \tag{1}$$

$$\frac{\partial}{\partial t}(\alpha_s \rho_s \boldsymbol{u}_s) + \nabla \cdot (\alpha_s \rho_s \boldsymbol{u}_s \boldsymbol{u}_s) = -\alpha_s \nabla p - \nabla p_s + \nabla \cdot \boldsymbol{\tau}_s + \alpha_s \rho_s \boldsymbol{g} + K_{gs}(\boldsymbol{u}_g - \boldsymbol{u}_s) \tag{2}$$

where g and s refer to gas and solid, $p$ is pressure, $\boldsymbol{\tau}_g$and $\boldsymbol{\tau}_s$ are stress tensors, $K_{gs}(\boldsymbol{u}_s - \boldsymbol{u}_g)$ is the drag term and $\nabla p_s$ is solid pressure. Note that there is a pressure gradient term in both equations and that it is multiplied with the volume fraction. Also gravity term is multiplied.

Now many classic drag laws are derived from experiments by assuming that there is a force balance between the gas pressure drop and drag (no acceleration or other forces):

$$\alpha_g \nabla p = K_{gs}(\boldsymbol{u}_s - \boldsymbol{u}_g) \tag{3}$$

For instance, the Ergun equation for pressure drop in dense beds is (ignoring non-sphericity):

$$\nabla p = 150 \frac{\alpha_s (1-\alpha_s)}{\alpha_g^3} \frac{\mu_g U_0}{d_p^2} + 1.75 \frac{\alpha_s}{\alpha_g^3} \frac{\rho_g U_0^2}{d_p}, \tag{4}$$

which can be substituted to (3) and we get the Ergun drag (note! superficial-velocity is $U_0 = \alpha_g(\boldsymbol{u}_g - \boldsymbol{u}_s)$ ):

$$K_{gs} = 150 \frac{\alpha_s (1-\alpha_s)}{\alpha_g} \frac{\mu_g}{d_p^2} + 1.75 \frac{\alpha_s \rho_g |\boldsymbol{u}_g - \boldsymbol{u}_s|}{d_p} \tag{5}$$

Alternatively, we can assume that the pressure drop exists only in the gas phase equations giving the following equations:

$$\frac{\partial}{\partial t}(\alpha_g \rho_g \boldsymbol{u}_g) + \nabla \cdot (\alpha_g \rho_g \boldsymbol{u}_g \boldsymbol{u}_g) = -\nabla p + \nabla \cdot \boldsymbol{\tau}_g + \rho_g \boldsymbol{g} + \widetilde{K}_{gs}(\boldsymbol{u}_s - \boldsymbol{u}_g) \tag{6}$$

$$\frac{\partial}{\partial t}(\alpha_s \rho_s \boldsymbol{u}_s) + \nabla \cdot (\alpha_s \rho_s \boldsymbol{u}_s \boldsymbol{u}_s) = -\nabla p_s + \nabla \cdot \boldsymbol{\tau}_s + \alpha_s (\rho_s - \rho_g) \boldsymbol{g} + \widetilde{K}_{gs}(\boldsymbol{u}_g - \boldsymbol{u}_s) \tag{7}$$

Note that there is no volume fraction in pressure gradient or gravity terms. Also there is no gas pressure gradient in solid momentum equation. The pressure effects are taken into account in drag term and buoyancy is added to solid phase gravity term. Before the applied patch, the gas momentum equation in MPPICFoam was like (6).

Now the force balance is (again no acceleration or other forces):

$$\nabla p = \widetilde{K}_{gs}(\boldsymbol{u}_s - \boldsymbol{u}_g) \tag{8}$$

We want to have the same pressure drop also here to match the experiments, so by comparing (3) and (8) we see that

$$\widetilde{K}_{gs} = \frac{K_{gs}}{\alpha_g} \tag{9}$$

In MPPICFoam the gas phase momentum equation has the same form as (6), so the drag models were divided with $\alpha_g$ to be consistent with the E-E approach.

The above equations were written for Eulerian-Eulerian models. When we convert a term, such as drag expression, from E-E context to a force acting on a particle, we write

$$F_{gs} = \frac{V_p}{\alpha_s} K_{gs}(\boldsymbol{u}_g - \boldsymbol{u}_s) = \frac{m_p}{\rho_p \alpha_s} K_{gs}(\boldsymbol{u}_g - \boldsymbol{u}_s)$$

So this is why there is (mass/p.rho()) in the drag models and why they are missing one $\alpha_s$ in addition to dividing with $\alpha_g$.

A discussion about whether to use equations [1-2] or [6-7] is given by Gidaspow [1]. He calls equations [1-2] “model A” and equations  [6-7] “model B”.

[1] Gidaspow, D. Multiphase Flow and Fluidization - Continuum and Kinetic Theory Descriptions. Academic Press, 1994. ISBN 0-12-282470-9.